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DESIGN METHOD OF CLOSED TEST SECTION

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Abstract: *A design method of the closed test section for subsonic wind tunnels is presented. The governing equation involved is the integro-differential form of the momentum equation. The method is applied to the rectangular and circular test section. A numerical example is achieved.*

***Keywords*:** Closed test section, Wind tunnel.

1. INTRODUCTION

One of the most important quality conditions of closed test section design is the absence of the longitudinal gradient of pressure. There are two methods to accomplish this condition:

a) The test section with solid walls has continuos increasing cross sections to cancel the effects of development of the boundary layer. Pankhurst recommends to design the walls flared with the displacement thickness δ_1 (Pankhurst, 1952). The common design practice uses 0.2^0 - $0.5^{0\ deg}$. of divergence for circular section, and up to 1^0 deg. for rectangular section (Marinescu, 1970). We notice that a single criterion for the calculation of flare angle does not exist.

b) Another solution is to use porous walls (Vaucheret, 1988). For two-dimensional subsonic wind tunnel of high speed, a suction window may be used to reduce sidewall boundary layer effect (Barnwell, 1993). Unfortunately this kind of test chamber is expensive, hard to construct and requires special apparatus.

The purpose of this paper is to suggest a new design method of the test section for subsonic wind tunnels. The method is based on the integro-differential form of the momentum equation written under the imposed criterion of a null longitudinal gradient of the pressure. This equation is then transformed into an ordinary differential equation: the cross-section equation, which involves only the boundary layer parameters δ_1 , δ_2 , c_f and the cross-section main parameters: area and perimeter (Popescu, 1998). Though developed for the rectangular and for the circular section, the method can be widely used to test sections of any form. Finally, a numeric example is performed.

2. BASIC HYPOTHESES

The suggested method is based on the following basic hypotheses:

(**H1**) *The test section is rectangular and constantly increasing to compensate the effects of the boundary layer* (Figure 1).

The static pressure would decrease if the section remained constant, due to the development of the boundary layer on the test chamber walls. Increasing this section compensates this tendency. The increasing section will have to ensure a constant static pressure along the test chamber (Pankhurst, 1952).

(**H2**) *The static pressure is constant in any cross-section of the test section*.

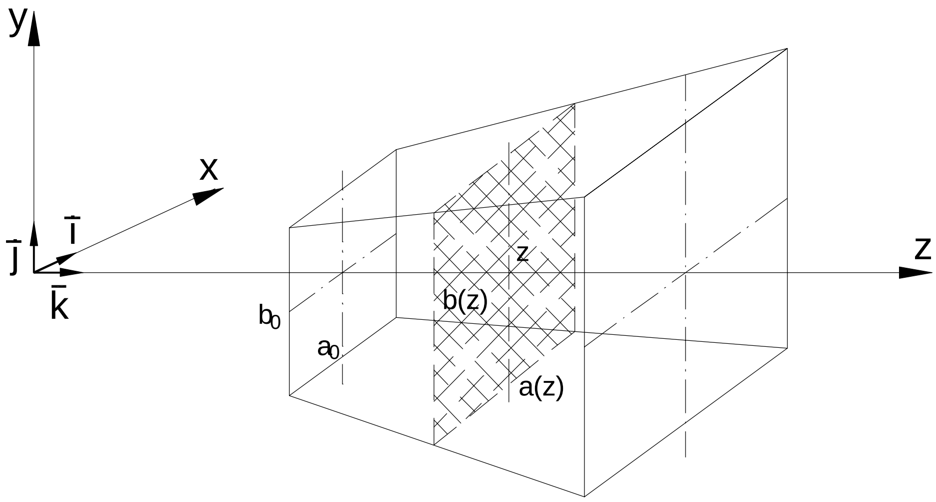

Figure 1 - Closed test section.

(**H3**) *The flow is of the constant velocity nucleus type*.

This hypothesis is justified by the presence of the contraction nozzle. The constant velocity nucleus occupies only the central part S_C (z) in any section (Fig.2). The peripheral area $S_\delta(z)$ is occupied by the boundary layer.

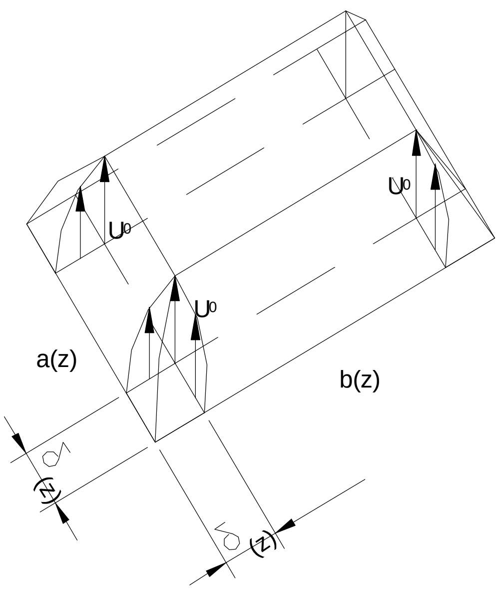

Figure 2- Velocity distribution in any cross-section of the test chamber

(**H4**) *We shall assume that in the section S(z) the boundary layer has the same thickness* $\delta(z)$, (Fig. 3).

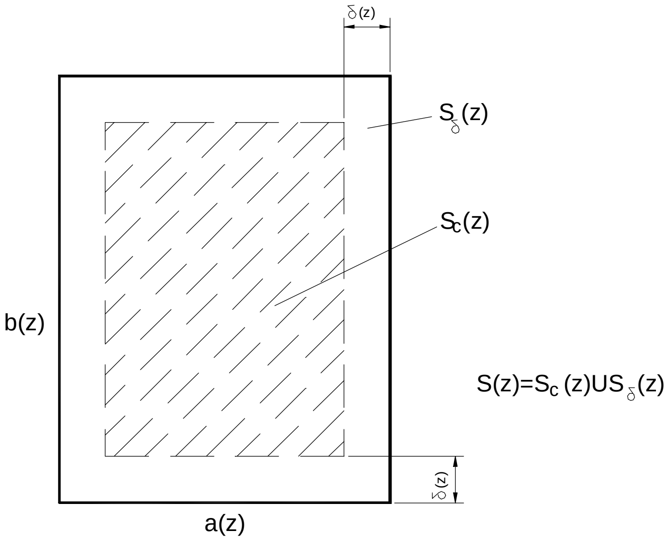

Figure 3- Notations for cross-section.

(**H5**) *The velocity distribution in the boundary layer will have a unidimensional form.* This will neglect the effect of the corners in which the distribution is bidimensional. In order to accomplish this hypothesis, the edged corners of the rectangular section must be cutted obliquely, or rounded.

The other hypotheses are secondary and will be introduced later.

3. THE CROSS-SECTION DIFFERENTIAL EQUATION

We separate the elementary domain (D) from the test section with help of the flux sections S(z), $S(z+\Delta z)$. The rigid boundary-surface of the tunnel wall is denoted with S_P (Fig.4).

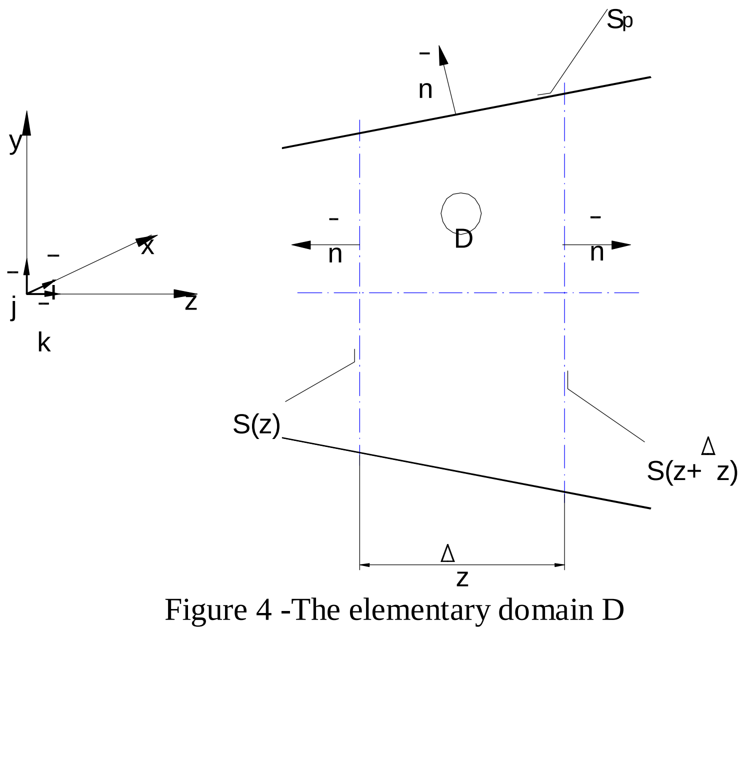

Figure 4 -The elementary domain D

We shall design the test section imposing a null longitudinal gradient of the pressure. According also to the (H2) hypothesis, we realize that the pressure is constant in the whole test section.

We shall assume the first supplementary hypothesis

(**H6**) *The pressure and density pulsations are negligible* $p' = 0 \quad \rho' = 0$. Then $p = \tilde{p}$ and $\rho = \tilde{\rho}$.

According to (H5), the momentum equation on the domain D has the following form

$$\int_{\partial D} \rho\, \tilde{u}_i \left(\tilde{u}_j n_j\right) d\sigma = \int_D \rho \cdot g_i\, dx + \int_{\partial D} \tilde{T}_{ij} \cdot n_j\, d\sigma \quad i=1,2,3 \tag{1}$$

where $\tilde{T}_{ij} = -\tilde{p}\delta_{ij} + \tilde{T}^V_{ij} + T^R_{ij}$. We have used the Einstein's convention. We will denote by $\tilde{T}_{ij}$ (or $\langle T_{ij} \rangle$) the turbulent-average values of the stresses; $\tilde{T}^V_{ij}$ are the average viscosity stresses; T^R_{ij} are the turbulence stresses.

Let us denote $u_3 = \tilde{u}_3$. Projecting Eq.1 according to $\bar{k} \equiv \bar{i}_3$ direction, and using the following condition $u'\big|_{Sp} = 0$, $T^R\big|_{Sp} = 0$, we obtain:

$$\begin{aligned} -\int_{S(z)} \rho u_3^2 d\sigma + \int_{S(z+\Delta z)} \rho u_3^2 d\sigma = & -\int_{S(z)} \left(\tilde{T}^V_{33} + T^R_{33}\right) d\sigma + \\ & + \int_{S(z+\Delta z)} \left(\tilde{T}^V_{33} + T^R_{33}\right) d\sigma - \int_{\partial D} p\, n_3 d\sigma + \int_{S_p} \left(\tilde{T}^V_{3j} n_j d\sigma\right) \end{aligned} \tag{2}$$

Applying the Gauss-Ostrogradski theorem, we find $\int_{\partial D} p\, n_3 d\sigma = \int_D \frac{\partial p}{\partial z} dx dy dz$. According to the design condition imposed, this integral will be null.

To derive from Eq.2 an integro-differential equation, we need the following results:

Proposition. *If* f *is a function of* C^1 *class, then*

$$\lim_{\Delta z \to 0} \frac{1}{\Delta z} \left(\int_{S(z+\Delta z)} f d\sigma - \int_{S(z)} f d\sigma \right) = \frac{d}{dz} \int_{S(z)} f d\sigma \tag{3}$$

$$\lim_{\Delta z \to 0} \frac{1}{\Delta z} \int_{S_p} f d\sigma = \int_{\partial S(z)} f \cdot \frac{1}{\cos\theta} ds \tag{4}$$

where, by definition, $\sin\theta = -\left(\bar{n}, \bar{k}\right)$.

Proof: The proof for eq.(3) is obvious. We notice that

$$\int_{S_p} f d\sigma = \int_z^{z+\Delta z} \left(\int_{\partial S(\zeta)} f(x, y, \zeta) \cdot \frac{1}{\cos\theta} ds \right) d\zeta$$

Hence, when $\Delta z \to 0$, we derive Eq.4.

Involving these two results, from Eq.(2) we derive the integro-differential form of momentum equation particularized for the test section

$$\frac{d}{dz}\int_{S(z)} \rho u_3^2 d\sigma = \frac{d}{dz}\int_{S(z)} \left(\tilde{T}_{33}^{v} + T_{33}^{R}\right) d\sigma + \int_{\partial S(z)} \tilde{T}_{3j}^{V} n_j \frac{1}{\cos\theta} ds \tag{5}$$

From the Bercker-Iacob lemma, we derive that $\left(\tilde{T}_{3j}^{V} n_j\right) = \tau_f \cdot \cos\theta$, where τ_f is the wall shear stress. We shall denote with A(z) the area of the cross-section S(z), and with P(z) the perimeter. For a rectangular section, we have

$$A(z) = \mu(S(z)) = a(z) \cdot b(z) \qquad\qquad P(z) = \mu(\partial S(z)) = 2(a(z) + b(z))$$

and using the hypothesis (H4), we derive

$$\int_{\partial S(z)} \tilde{T}_{3j}^{V} n_j \frac{1}{\cos\theta} ds = \tau_f(z) \cdot P(z) \tag{6}$$

According to the hypothesis (H3) we have

$$u_3(x, y, z) = \begin{cases} U_0 & (x, y) \in S_c(z) \\ u(x, y) & (x, y) \in S_\delta(z) \end{cases} \tag{7}$$

where u(x,y) is a velocity distribution of the boundary layer type. It results:

$$\int_{S(z)} u_3^2 d\sigma = U_0^2 \cdot \left(A(z) - \int_{S_\delta(z)} \left(1 - \frac{u^2}{U_0^2}\right) d\sigma \right)$$

To calculate the integral, according to (H6), we shall "develop" the perimetral area $S_\delta(z)$ and we shall approximate it with a rectangle of the sides P(z) and δ(z). Thus, after an elementary calculus, we obtain

$$\int_{S_\delta(z)} \left(1 - \frac{u^2}{U_0^2}\right) d\sigma = P(z) \cdot \int_0^{\delta(z)} \left(1 - \frac{u^2(y)}{U_0^2}\right) dy = P(z) \cdot \left(\delta_1(z) + \delta_2(z)\right) \tag{8}$$

where $\delta_1(z) = \int_0^{\delta} (1 - \frac{u}{U_0}) dy$ is the displacement thickness and $\delta_2(z) = \int_0^{\delta} \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy$ is the momentum-loss thickness.

According to the same hypothesis, we find

$$\frac{d}{dz}\int_{S(z)}\left(\tilde{T}_{33}^{v}+T_{33}^{R}\right)d\sigma=\frac{d}{dz}\left[P(z)\cdot\int_{0}^{\delta(z)}\left(\tilde{T}_{33}^{v}+T_{33}^{R}\right)dy\right]$$

Replacing in Eq.(5) we derive the differential equation of the test cross-section

$$\begin{aligned}&\frac{dA}{dz}(z)-\frac{d}{dz}\left[P(z)\cdot\left(\delta_1(z)+\delta_2(z)\right)\right]=\\&\frac{d}{dz}\left[P(z)\cdot\int_{0}^{\delta(z)}\frac{1}{U_0^2}\left(\tilde{T}_{33}^{v}+T_{33}^{R}\right)dy\right]-\frac{1}{2}c_f(z)\cdot P(z)\end{aligned}\tag{9}$$

where $c_f=-2\tau_f/\rho U_0^2$ is the friction coefficient.

We notice that

$$\begin{aligned}&\frac{d}{dz}\left[P(z)\cdot\frac{1}{U_0^2}\int_{0}^{\delta(z)}\left(\tilde{T}_{33}^{v}+T_{33}^{R}\right)dy\right]=\\&\left[\frac{dP}{dz}\cdot\int_{0}^{\delta(z)}\left(\frac{1}{U_0^2}\tilde{T}_{33}^{v}+\frac{1}{U_0^2}T_{33}^{R}\right)dy+P(z)\cdot\frac{d}{dz}\left(\frac{1}{U_0^2}\tilde{T}_{33}^{v}+\frac{1}{U_0^2}T_{33}^{R}\right)\left(\delta(z)\right)\cdot\frac{d\delta}{dz}\right]\end{aligned}$$

and, in a first approximation, we shall neglect this term. Eq. (9) becomes

$$\frac{dA}{dz}(z)-\frac{d}{dz}\left[P(z)\cdot\left(\delta_1(z)+\delta_2(z)\right)\right]=-\frac{1}{2}c_f(z)\cdot P(z)\tag{9'}$$

3. THE CHARACTERISTIC PARAMETERS OF THE BOUNDARY LAYER

We shall assume the second supplementary hypothesis:

(**H7**) *The boundary layer is turbulent even from the beginning,* due to the vortex generators.

We shall consider a logarithmic velocity distribution in the boundary layer:

$$u(y,z)=u_f(z)\cdot\left[A\cdot\ln\frac{y}{y_f(z)}+B\right]\tag{10}$$

According to the Coles model for smooth walls, A=2,5 and B=5 (Reynolds, 1974). We denote with $u_f(z)$ the wall-friction velocity $u_f(z)=\sqrt{|\tau_f(z)|/\rho}$, and with y_f the reference length $y_f=\nu/u_f$.

From the match condition $U_0 = u_f(z) \cdot \left[A \cdot \ln \frac{\delta(z)}{y_f(z)} + B \right] \quad (\forall) z$, eq.(8) become $1 - \frac{u(y,z)}{U_0} = \frac{u_f(z)}{U_0} A \ln \frac{\delta(z)}{y}$. By replacing in the above expressions the displacement thickness and the momentum-loss thickness, we obtain (Reynolds, 1974):

$$\frac{\delta_1(z)}{\delta(z)} = A \sqrt{\frac{c_f(z)}{2}} \tag{11}$$

$$\frac{\delta_2(z)}{\delta(z)} = A \sqrt{\frac{c_f(z)}{2}} \cdot \left[1 - 2A \sqrt{\frac{c_f(z)}{2}} \right] \tag{12}$$

Schlichting has suggested a formula for the friction coefficient, which gives very good results in the domain $10^5 < Re_L < 10^9$:

$$c_f = \left(2 \log_{10} Re_z - 0{,}65 \right)^{-2{,}3} \tag{13}$$

where $Re_L = U_0 \cdot L / \nu$, $\quad Re_z = U_0 \cdot z / \nu$.

To these expressions, one adds up (Schlichting, 1987):

$$\frac{1}{2} c_f(z) = \frac{d\delta_2}{dz} \tag{14}$$

4. THE CASE OF THE RECTANGULAR SECTION

We impose a uniform increase of the cross-section sides

$$a(z) = a_0 + 2g(z) \qquad\qquad b(z) = b_0 + 2g(z) \tag{15}$$

Then $A(z) = a_0 b_0 + 2(a_0 + b_0) g(z) + 4g^2(z)$, $\quad P(z) = 2(a_0 + b_0) + 8g(z)$.

The length of the test chamber will be denoted with L. We introduce the following dimensionless quantities

$$\bar{z} = \frac{z}{L}, \bar{z} \in [0,1]; \quad \bar{a}_0 = \frac{a_0}{L}; \quad \bar{b}_0 = \frac{b_0}{L}; \quad \bar{g}(\bar{z}) = \frac{g(z)}{L}; \quad \bar{\delta}_1 = \frac{\delta_1}{L}; \quad \bar{\delta}_2 = \frac{\delta_2}{L} \tag{16}$$

It results $\frac{dA}{dz} = 2L \frac{d\bar{g}}{d\bar{z}} \left(\bar{a}_0 + \bar{b}_0 + 4\bar{g} \right)$ and $\frac{dP}{dz} = 8 \frac{d\bar{g}}{d\bar{z}}(z)$.

Replacing these relations in eq.(9’) and using eq.(14), we derive

$$\frac{d\bar{g}}{d\bar{z}} = \frac{\left(\frac{1}{4} \left(\bar{a}_0 + \bar{b}_0 \right) + \bar{g} \right) \cdot \frac{d\bar{\delta}_1}{d\bar{z}}}{\frac{1}{4} \left(\bar{a}_0 + \bar{b}_0 \right) + \bar{g} - \left(\bar{\delta}_1 + \bar{\delta}_2 \right)} \tag{17}$$

We add the following equations

$$\frac{\bar{\delta}_2(\bar{z})}{\bar{\delta}_1(\bar{z})}=\left[1-2A\sqrt{\frac{1}{2}\left(2\log_{10}\bar{z}\,\mathrm{Re}_L-0{,}65\right)^{-2{,}3}}\right] \tag{18}$$

$$\frac{d\bar{\delta}_2}{d\bar{z}}=\frac{1}{2}\left(2\log_{10}\bar{z}\,\mathrm{Re}_L-0{,}65\right)^{-2{,}3} \tag{19}$$

and the initial condition

$$\bar{g}(0)=0 \tag{20}$$

The eq.(17), (18), (19), and (20) form an initial-value problem. Because the implied functions are supposed to be of C^1 class, the local existence and uniqueness of the solution are ensured.

5. THE CASE OF THE CIRCULAR SECTION

We also impose a uniform increase of the cross-section

$$R(z)=R_0+g(z) \tag{21}$$

Then $A(z)=\pi\cdot\left(R_0+g(z)\right)^2$ and $P(z)=2\cdot\pi\cdot\left(R_0+g(z)\right)$. Following the same procedure, we derive the circular cross-section equation

$$\frac{d\bar{g}}{d\bar{z}}=\frac{\left(\bar{R}_0+\bar{g}\right)\cdot\frac{d\bar{\delta}_1}{d\bar{z}}}{\bar{R}_0+\bar{g}-\left(\bar{\delta}_1+\bar{\delta}_2\right)} \tag{22}$$

where $\bar{R}_0=R_0/L$.

6. RESULTS AND DISCUSSIONS

1) We have done a computation for a rectangular test chamber with the dimensions: a_0=0.140m; b_0 =0.290m, L=0.5m. The corners are cutted obliquely at $(1/7)\cdot a_0$. The flow parameters have the following values: U_0=90m/s; p_0=101000 Pa; $\nu=15*10^{-6}\,m^2/s$. We denote by L_0 the length of the antechamber and we assume that the effective origin of the boundary layer is 30 mm before the antechamber.

The differential equation has been numerically solved with the Runge-Kutta method of the 4-th order, improved by Gill. A correction with the 4-th order Adams-Moulton scheme was done finally (Ixaru, 1979).

We shall denote by $\alpha=\arctan\left(g(L)/L\right)$ the half-divergence angle and by $\Delta\delta_1(L)=\delta_1(L)-\delta_1(0)$ the Pankhurst's design criterion.

The results are graphically and numerically presented in Fig. 5 and in Table 1.

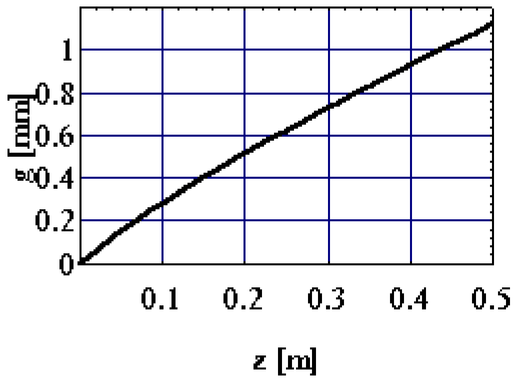

Figure 5- g(z) variation along the rectangular test section; L0=0

Tab. 1

L_0 [mm]	0	50	100	150	200	250
α [deg]	0.128	0.122	0.118	0.115	0.112	0.110
g(L) [mm]	1.123	1.067	1.031	1.004	0.982	0.964
$\Delta\delta_1$ [mm]	1.113	1.053	1.017	0.989	0.966	0.946

b) From (17), it results $\frac{dg}{dz} > \frac{d\delta_1}{dz}$. Involving the differential inequality theorem, we derive that $g(z) > \delta_1(z) - \delta_1(0) \quad (\forall) z \in [0, L]$. This trend was numerically confirmed (see Tab.1).

c) The problem concerning the boundary layer's parameters in the exit zone of the contraction cone was avoided by assignation of the origin of the turbulent boundary layer at -L_0-0.03. A significant improvement, as to the applicability of the method, will be achieved by supplementary experimental data regarding the boundary layer's evolution in the above mentioned zone.

d) The method may be adapted as well for other regular shapes of cross-section, too. In all cases for which the hypothesis (H4) is fulfilled (the corner effects may be neglected), eq.(9) remains unmodified. Also, the motion regime (Re number) may change only the auxiliary eq. (18) and (19). For both these reasons, we can see eq. (9) like a universal equation. This can be considered the main advantage of this method.

For comparison, numerical calculations were done for a circular section, too with the same initial area $\pi \cdot R_0^2 = a_0 \cdot b_0$. The results are practically the same with those obtained for the rectangular section with cutted corners. For a rectangular section with edged corners, this proximity should be expected no more.

The values obtained for the half-divergence angle α are lying in the interval recommended by the common design practice.

e) The eq. (9') was obtained as a result of an approximation, by neglecting some small terms (for instance, $\tilde{T}_{33}^{V} = (\mu_2 + 2\mu_1)\frac{\partial u}{\partial y} \cong 1 \cdot 10^{-5} \cdot \frac{\partial u}{\partial y}$). To increase the computation precision, thus getting closer to the exact solution, the problem may be considered as being of successive approximations. Thus after solving the initial value problem (17)+(20) we shall determine at a first approximation the values of $\frac{dP}{dz}(z)$ and $P(z)$. Others, such as, for instance $(\mu_2 + 2\mu_1)\frac{\partial u}{\partial y}$, $\rho \cdot \langle u'u' \rangle$, will be estimated on the basis of the experimental data found in literature (Cebeci, 1974).

For other terms, such $\frac{d}{dz}\left(\frac{1}{U_0^2}\tilde{T}_{33}^{v} + \frac{1}{U_0^2}T_{33}^{R}\right)(\delta(z))$, the estimation will be a difficult task. The experimental data presented in specialized papers are reported on variations according to the normal direction. Only a few experimental data regarding the longitudinal variations are available. Especially for this reason, a qualitative analysis of the small parameter's influences on the solution is required. We will focus on this subject in a future paper.

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